An Extension of Fractional Parentage Expansion to the Nonrelativistic and Relativistic $SU^f(3)$ Dibaryon Calculations

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Abstract

The fractional parentage expansion method is extended from $SU^f(2)$ nonrelativistic to $SU^f(3)$ and relativistic dibaryon calculations. A transformation table between physical bases and symmetry bases for the $SU^f(3)$ dibaryon is provided. A program package is written for dibaryon calculation based on the fractional parentage expansion method.

I. INTRODUCTION

Quantum chromodynamics (QCD) is a very promising theory for fundamental strong interaction. However, due to the complexity of QCD, for the present time and for the foreseeable future, one must rely on QCD inspired models to study hadron physics. The existing models (potential, bag, soliton, etc.) are quite successful for the meson and baryon sectors, but not so successful for hadronic interactions. Recently some hope has developed to obtain the full N-N interaction from QCD models[1,2].

Since the first prediction of H particle by Jaffe[3], there have been tremendous efforts both theoretically and experimentally[4] to find possible candidates for quasi-stable dibaryon states. Nevertheless, there remains an outstanding question. Theoretically all the QCD models, including lattice QCD calculations, predict that there should be quasi-stable dibaryons or dibaryon resonances, but in contrast, experimentally no quasi-stable dibaryon whatsoever has been observed (except the molecular deuteron state). One has to ask if some important QCD characteristics are missing in all these dibaryon calculations. For example in the potential (or cluster) model approach, the six quark Hamiltonian is usually a direct extension of the three quark Hamiltonian. This extension is neither reasonable nor successful. The two-body confinement potential yields color van der Waals forces which are in contradiction with experimental observation. Lattice gauge calculations and nonperturbative QCD both yield a string-like structure inside a hadron instead of two-body confinement. Two-body confinement may be a reasonable approximation inside a hadron but not for the interaction between quarks in two color singlet hadrons[5]. Another possible missed general feature is that the quark, originally confined in a single hadron, may tunnel (or percolate) to the other hadron when two hadrons are close together[6]. In the potential model approach, the internal motion of the interacting hadrons is assumed to be unchanged. The product ansatz of the Skyrmion model approach makes the same approximation. In the bag model approach, another extreme approximation is assumed, *i.e.*, the six quark are merged into a single confinement space. The real configuration may be in between these two extremes which is well known in the molecular physics.

Except for a few cluster model calculations, in which a phenomenological meson exchange ls involved to fit the N-N scattering, for all the other dibaryon calculations, the model parameters are only constrained by hadron spectroscopy. In fact, the six quark system includes new color structures, for which a single hadron cannot give any information. A six quark Hamiltonian should be constrained by the existing baryon-baryon interaction data, especially the N-N data, then the model dibaryon states may be really relevant to the experimental measurement.

A model, the quark delocalization color screening model(QDCSM), has been de-

veloped which includes the new QCD inspired ingredients mentioned above and is constrained by N-N scattering data[2]. This model has been applied to a systematic search of the dibaryon candidates in the u d and s three flavor world[7] to provide a better estimate of dibaryon states on the one hand and to test the model assumption further on the other hand.

As pointed out in [4], a more realistic systematic search of dibaryons would be a tremendous task, for which a systematic and powerful method is indispensable. The fractional parentage (fp) expansion developed in atomic and nuclear physics is one of such methods. A major obstacle in applying the fp expansion technique to quark models is the occurrence of many $SU(mn) \supset SU(m) \times SU(n)$ isoscalar factors(ISF) with $m, n \geq 2$. e.g., in the two "orbits", two spins, n_f flavors and 3 colors quark world. We need the $SU(2\times 3\times n_f \times 2) \supset SU^x(2)\times SU(6n_f), SU(6n_f) \supset SU^c(3)\times SU(2n_f), SU(2n_f) \supset SU^f(n_f)\times SU^\sigma(2)$ ISF 's, where x, c, f and σ indicate the space or orbit, color, flavor and spin respectively. Before 1991, only the $SU(4) \supset SU(2) \times SU(2)$ ISF (and some scattered results for $SU(6) \supset SU(3) \times SU(2)$ ISF were available. A breakthrough in group representation theory is the recognition of the fact that the $SU(n_1 n_2) \supset SU(n_1)\times SU(n_2)$ n_2-particle coefficients of $fp(cfp)$ are precisely the ISF for the permutation group chain $S(n_1+n_2) \supset S(n_1)\times S(n_2)$[8a], and the former can be calculated and tabulated in a rank independent way, instead one m and n at a time. In 1991, Chen *et al*[8b] published a book with phase consistent $SU(mn) \supset SU(m) \times SU(n)$ ISF for arbitrary m and n and for up to six particles. Because of this, we are now in a position to develop an efficient algorithm for dibaryon calculations based on the fp technique. This paper reports the extension of the fp expansion to the nonrelativistic and relativistic $SU^f(3)$ quark model calculation in line with the work of Harvey[9] and of Chen[l0].

II. PHYSICAL BASES AND SYMMETRY BASES

A dibaryon may be a loosely bound two q^3 cluster state like the deuteron, or it may be a tightly bound q^6 cluster like the Jaffe's version of the H particle. Many cases may be in between. To describe these states, the physical basis is preferable due to its apparent dibaryon content in the asymptotic region without artificial confinement assumptions. The physical basis is nothing else but the cluster model basis developed in the nuclear cluster model[11]. To show the symmetry property explicitly, we follow Chen's notation[8,10] but with a slight modification, because we are working in the u, d, s three flavor world instead of u, d flavors.

A baryon in the u, d, s three flavor world is described by

$$\psi(B) = \left| [\sigma] W \overset{[\nu]}{[\mu]} [f] Y I J \right\rangle , \tag{1}$$

which is a basis vector belonging to the irreducible representations (irreps)

$$\begin{aligned} \overset{[1^3]}{SU(36)} \supset & \left(\overset{[\nu]}{SU^x(2)} \times \left(\overset{[\tilde{\nu}]}{SU(18)} \supset \overset{[\sigma]}{SU^c(3)} \times \left(\overset{[\mu]}{SU(6)} \supset \right.\right.\right. \\ & \left.\left.\left. \left(\overset{[f]}{SU^f(3)} \supset \overset{I}{SU^\tau(2)} \times \overset{Y}{U^Y(1)} \right) \times \overset{J}{SU^\sigma(2)} \right) \right) \right) , \end{aligned} \tag{2}$$

where the first reduction is to orbital times combined color-flavor-spin symmetry, the second reduces the latter to color times combined flavor-spin, and the third reduces the last to flavor (which is itself reduced to isospin times hypercharge) times spin. Here $[\nu]$ etc. are the Young diagrams describing the permutational and $SU(n)$ symmetries. In our calculation, the ground state baryons are assumed to be in the totally symmetric orbital state $[\nu] = [3]$, while $[\sigma] = [1^3]W$ is the Weyl tableau for the $SU^c(3)$ state due to color confinement, i.e., the baryon is colorless. On the other hand, $[\mu] = [3]$ due to the totally antisymmetry requirement $[\sigma] \times [\mu] \to [\tilde{\nu}] = [1^3], [\tilde{\nu}]$ being the conjugate Young diagram of $[\nu]$. $[f]$ and $[\sigma_J]$ are restricted by the condition $[f] \times [\sigma_J] \to [\mu] = [3]$, this leads to $[f] = [\sigma_J] = [3]$ or $[21], [\sigma_J]$ represents the spin symmetry, $[\sigma_J] = [\frac{n}{2} + J, \frac{n}{2} - J], n$ is the total number of quarks. i.e., the $SU^f(3)$ decuplet and octet baryons $\Delta, \Sigma^*, \Xi^*, \Omega$ and N, Λ, Σ, Ξ. The symbols Y, I, J denote the hypercharge, isospin and spin quantum numbers respectively. M_I and M_J, the magnetic quantum numbers, are omitted in eq.(1).

A two baryon physical basis is described by

$$\begin{aligned} \Psi_{\alpha k}(B_1 B_2) &= \mathcal{A}[\psi(B_1)\psi(B_2)] \overset{[\sigma]}{W} \overset{I}{M_I} \overset{J}{M_J} \\ &= \mathcal{A} \left[\left| [\sigma_1][\mu_1]\overset{[\nu_1]}{[f_1]} Y_1 I_1 J_1 \right\rangle \left| [\sigma_2][\mu_2]\overset{[\nu_2]}{[f_2]} Y_2 I_2 J_2 \right\rangle \right]^{[\sigma]\ I\ \ J}_{W M_I M_J} , \end{aligned} \tag{3}$$

here $[\ \ ]^{[\sigma]\ I\ \ J}_{W M_I M_J}$ means the couplings in terms of the $SU^c(3), SU^\tau(2)$ and $SU^\sigma(2)$ Clebsch-Gordan coefficients (CGC) so that it has the total color symmetry $[\sigma]W$, isospin IM_I and spin JM_J. Due to color confinement, only the overall color singlet $[\sigma] = [2^3]$ is allowed. $\mathcal{A}$ is a normalized antisymmetric operator. $\alpha = (YIJ)$ with $Y = Y_1 + Y_2, k$ represents the quantum numbers $\nu_i, \sigma_i, \mu_i, f_i, I_i, J_i (i = 1, 2)$.

To take into account the mutual distortion or the internal orbital excitation of the interacting baryons when they are near one another, the delocalized single quark

state $l(r)$ is used for baryon $B_1(B_2)$[2, 6, 7]

$$l = (\phi_L + \epsilon(s)\phi_R)/N(s), r = (\phi_R + \epsilon(s)\phi_L)/N(s)$$
$$\phi_L(\vec{r}) = \left(\frac{1}{\pi b^2}\right)^{\frac{3}{4}} e^{-\frac{(\vec{r}-\vec{s}/2)^2}{2b^2}}, \tag{4}$$
$$N^2(s) = 1 + \epsilon^2(s) + 2\epsilon(s)\langle\phi_L|\phi_R\rangle.$$

The $\vec{s}$ is the separation between two q^3 cluster centers, $\epsilon(s)$ is a parameter describing the delocalization (or percolation) effect, and it is determined variationally by the q^6 dynamics. Hidden color channels are not included in eq.(3), because it has been proven[12] that the colorless hadron channels form a complete Hilbert space if the excited colorless baryon states are included. Also the concept of a colorful hadron has not been well defined in QCD models.

Physical bases are not convenient for matrix element calculations. To take advantage of the fp expansion technique developed in atomic and nuclear physics, one has to use symmetry bases (group chain classification bases). This requires an extension of the q^3 state eq.(1) to the q^6 case,

$$\Phi_{\alpha K}(q^6) = \left| [\sigma]W[\mu]\beta\overset{[\nu]l^3r^3}{[f]}YIJM_IM_J \right\rangle. \tag{5}$$

Here K represents the quantum numbers $[\nu], [\mu], \beta, [f]$ appeared in eq.(5). $[\sigma] = [2^3]$ due to color confinement. To be consistent with the physical basis choice, the orbital part is truncated to include the l^3r^3 configuration only. $[\nu]$ is restricted to be $[\nu] = [3] \times [3] = [6] + [51] + [42] + [33]$. β is the inner multiplicity index in the reduction $[\mu] \to [f] \times [\sigma_J]$.

Physical and symmetry bases both form a complete set in a truncated Hilbert space, and are related by a unitary transformation. Harvey[9] first calculated the transformation coefficients for the u, d two flavor case. Chen[10] proved that the transformation coefficients are just a product of $(6 \to 3+3)SU(mn) \supset SU(m) \times SU(n)$ isoscalar factors. Here we extend them to the $SU^f(3)$ case,

$$\mathcal{A}\left[\left| [\sigma_1][\mu_1]\overset{[\nu_1]l^3}{[f_1]}Y_1I_1J_1 \right\rangle \left| [\sigma_2][\mu_2]\overset{[\nu_2]r^3}{[f_2]}Y_2I_2J_2 \right\rangle\right]^{[\sigma]\ I\ \ J}_{WM_IM_J} \tag{6}$$
$$= \sum_{\tilde{\nu}\mu\beta f\gamma} C^{[\tilde{\nu}][\sigma][\mu]}_{[\tilde{\nu}_1][\sigma_1][\mu_1][\tilde{\nu}_2][\sigma_2][\mu_2]} C^{[\mu]\beta[f]\gamma J}_{[\mu_1][f_1]J_1,[\mu_2][f_2]J_2} C^{[f]\gamma YI}_{[f_1]Y_1I_1[f_2]Y_2I_2} \left| {}^{[\nu]l^3r^3}_{[\sigma]W[\mu]\beta[f]YIJM_IM_J} \right\rangle.$$

This expression is written simply as

$$\Psi_{\alpha k}(B_1B_2) = \sum_K C_{kK}\Phi_{\alpha K}(q^6), \tag{7}$$

here γ is an outer multiplicity index in the reduction $[f_1] \times [f_2] \to [f]$. The first two C factors in Eq.(6) are $SU(18) \supset SU^c(3) \times SU(6)$ and $SU(6) \supset SU^f(3) \times SU^\sigma(2)$ isoscalar factors respectively and the third one is $SU^f(3) \supset SU^\tau(2) \times U^Y(1)$ isoscalar factor. All these isoscalar factors can be found in Chen's book[8b]. The calculated transformation coefficients are listed in table 1. The $Y = 2$ part is a revised version (phase consistent and simplified for $I = J = 1$ case) of Harvey's table 11[9]. (The relationship between our Tables and those of Harvey, is discussed in the Appendix.) The $Y \neq 2$ part is an extension of Harvey's two flavor case to three flavor case. Because the hidden color channels are not included, this table can be used to expand the physical bases in terms of the symmetry bases only. If one wants to expand the symmetry bases in terms of the physical bases, then the hidden color physical bases (or other equivalent set of bases) should be added. One example is given below,

$$
\begin{aligned}
|\, H\,\rangle \;\; &= \;\; \sqrt{\frac{1}{5}}\left(\sqrt{\frac{3}{8}}\,|\Sigma\Sigma\rangle - \sqrt{\frac{4}{8}}\,\left|\bar{N\Xi}\right\rangle - \sqrt{\frac{1}{8}}|\Lambda\Lambda >\right) \\
&- \;\; \sqrt{\frac{6}{40}}\,|\Sigma\Sigma\rangle_c + \sqrt{\frac{8}{40}}\,\left|\bar{N\Xi}\right\rangle_c + \sqrt{\frac{2}{40}}\,|\Lambda\Lambda\rangle_c \\
&+ \;\; \sqrt{\frac{3}{40}}\,|\Sigma'\Sigma'\rangle_c - \sqrt{\frac{4}{40}}\,\left|\bar{N'\Xi'}\right\rangle_c - \sqrt{\frac{1}{40}}\,|\Lambda'\Lambda'\rangle_c - \sqrt{\frac{8}{40}}\,|\Lambda_s\Lambda_s\rangle_c\,. \qquad (8)
\end{aligned}
$$

Here $|\overline{XY}\}$ means the symmetric channel of baryons X and Y, $|XY\rangle_c$ means hidden color channel of colorful baryons X and Y, Λ_s is the flavor singlet Λ, X' represents excited colorful baryon with spin $\frac{3}{2}$. In the prevailing literature, only first three colorless channels are given[13]. See the Appendix for a description of the difference between our meaning for symmetry and that of Harvey [9].

III FRACTIONAL PARENTAGE EXPANSION

A physical six quark state with quantum number $\alpha = (YIJ)$ is expressed as a channel coupling wave function (WF)

$$\Psi_\alpha = \sum_k C_k \Psi_{\alpha k}(B_1 B_2). \qquad (9)$$

The channel coupling coefficients C_k are determined by the diagonalization of the six quark Hamiltonian as usual. To calculate the six quark Hamiltonian matrix elements in the physical basis,

$$H_{kk'} = \langle \Psi_{\alpha k} || H || \Psi_{\alpha k'} \rangle\,, \qquad (10)$$

is tedious. We first express the physical basis in terms of the symmetry basis by the transformation eq.(6), and the matrix element eq.(l0) is transformed into a sum of matrix elements in the symmetry basis

$$H_{kk'} = \sum_{K,K'} C_{kK} C_{k'K'} \left\langle \Phi_{\alpha K} | H | \Phi_{\alpha K'} \right\rangle . \tag{11}$$

The matrix elements $\{\Phi_{\alpha K}|H|\Phi_{\alpha K'}\}$ can be calculated by the well known fp expansion method,

$$\begin{aligned} \left\langle \Phi_{\alpha K} | H | \Phi_{\alpha K'} \right\rangle = \sum \binom{6}{2} \left\langle \Phi_{\alpha K} | \alpha_1 K_1, \alpha_2 K_2 \right\rangle \left\langle \alpha_1' K_1', \alpha_2' K_2' | \Phi_{\alpha K'} \right\rangle \left\langle \alpha_1 K_1 | \alpha_1' K_1' \right\rangle \\ \left\langle \alpha_2 K_2 | H_{56} | \alpha_2' K_2' \right\rangle . \end{aligned} \tag{12}$$

Here $\langle \alpha_1 K_1 | \alpha_1' K_1' \rangle$ is the four quark overlap, and is a little more complicated than the atomic and nuclear shell model case due to the non-orthogonal property of the single quark orbital state (see below). $\langle \alpha_2 K_2 | H_{56} | \alpha_2' K_2' \rangle$ is the two body matrix element and H_{56} represents the two-body operator for the last pair. $\binom{6}{2} = 15$ is the interacting pair number. To simplify the computer program, the one-body operator matrix elements are calculated by the same expansion eq.(12) with the obvious substitution $H_{56} \rightarrow H_5 + H_6$ and $\binom{6}{2} \rightarrow \frac{6}{2} = 3$ (only six one-body operators altogether instead of 15 pair interactions).

$\langle \Psi_{\alpha K} | \alpha_1 K_1, \alpha_2 K_2 \rangle$ and $\langle \alpha_1' K_1', \alpha_2' K_2' | \Phi_{\alpha K'} \rangle$ are the total Clebsch-Gordon Coefficients (CGC). They are calculated as follows[8b].

$$\begin{aligned} &\left\langle [\sigma] W [\mu] \beta \overset{[\nu] l^3 r^3}{[f]} Y I J M_I M_J \middle| [\sigma_1] W_1^c, [\mu_1] \overset{[\nu_1] W_1^x}{[f_1]} Y_1 I_1 J_1 M_{I_1} M_{J_1}, [\sigma_2] W_2^c [\mu_2] \overset{[\nu_2] W_2^x}{[f_2]} Y_2 I_2 J_2 M_{I_2} M_{J_2} \right\rangle \\ &= \sum_{\gamma} C^{[\sigma]W}_{[\sigma_1]W_1^c, [\sigma_2]W_2^c} \, C^{IM_I}_{I_1 M_{I_1}, I_2 M_{I_2}} \, C^{JM_J}_{J_1 M_{J_1}, J_2 M_{J_2}} \, C^{[\nu] l^3 r^3}_{[\nu_1] W_1^x, [\nu_2] W_2^x} \\ &\quad C^{[1^6][\nu][\tilde{\nu}]}_{[1^4][\nu_1][\tilde{\nu}_1],[1^2][\nu_2][\tilde{\nu}_2]} \, C^{[\tilde{\nu}][\sigma][\mu]}_{[\tilde{\nu}_1][\sigma_1][\mu_1],[\tilde{\nu}_2][\sigma_2][\mu_2]} \, C^{[\mu]\beta[f]\gamma J}_{[\mu_1][f_1][J_1],[\mu_2][f_2][J_2]} \, C^{[f]\gamma Y I}_{[f_1]Y_1 I_1,[f_2]Y_2 I_2} \quad . \end{aligned} \tag{13}$$

The first four C's are the $SU^c(3), SU^\tau(2)$, $SU^\sigma(2)$ and $SU^x(2)$ CGC, the next three C's are the $SU(36) \supset SU^x(2) \times SU(18), SU(18) \supset SU^c(3) \times SU(6), SU(6) \supset SU^f(3) \times SU^\sigma(2)$ isoscalar factors, the last one is the $SU^f(3) \supset SU^\tau(2) \times U^Y(1)$ isoscalar factor. All these isoscalar factors (for particle number ≤ 6) can be found in ref.[8b]. The $SU^x(2)$ orbital CGC $C^{[\nu]l^3 r^3}_{[\nu_1]W_1^x,[\nu_2]W_x^x}$ is called the orbital two-body cfp by Harvey and listed in his table 4[9]. It is obvious that it is better to use the standard phase convention of the SU(2) CGC. Then the entries under [4] : [ll] $a^2b^2 : \tilde{ab}$, [31] : [2] $ab^3 : a^2$ and [31] : [2]$a^2b^2 : \bar{ab}$ should be assigned opposite signs.

The four-quark state $|\alpha_1 K_1\rangle$ can be expressed as[10]

$$|\alpha_1 K_1\rangle = \left| [\sigma_1] W_1^c [\mu_1][f_1] \overset{[\nu_1]W_1^x}{Y_1} I_1 J_1 M_{I_1} M_{J_1} \right\rangle \tag{14}$$

$$= \sum_m (h_{\nu_1})^{-\frac{1}{2}} \Lambda_m^{\nu_1} \left| \begin{smallmatrix} [\nu_1]W_1^x \\ m \end{smallmatrix} \right\rangle \left| \begin{smallmatrix} [\tilde{\nu}_1] \\ \tilde{m} \end{smallmatrix} [\sigma_1] W_1^c [\mu_1][f_1] Y_1 I_1 M_{I_1} M_{J_1} \right\rangle ,$$

here $m(\tilde{m})$ is the Yamanouchi number of the Young tableau, $(h_{\nu_1})^{-\frac{1}{2}} \Lambda_m^{\nu_1}$ is the CGC for $[\nu_1] \times [\tilde{\nu}_1] \to [1^4]$ of the permutation group. The color-flavor-spin part

$$\left| \begin{smallmatrix} [\tilde{\nu}_1] \\ \tilde{m} \end{smallmatrix} [\sigma_1] W_1^c [\mu_1][f_1] Y_1 I_1 J_1 M_{I_1} M_{J_1} \right\rangle$$

is orthogonal as usual

$$\left\langle \begin{smallmatrix} [\tilde{\nu}_1] \\ \tilde{m} \end{smallmatrix} [\sigma_1] W_1^c [\mu_1][f_1] Y_1 I_1 J_1 M_{I_1} M_{J_1} \middle| \begin{smallmatrix} [\tilde{\nu}_1'] \\ \tilde{m}' \end{smallmatrix} [\sigma_1'] W_1'^c [\mu_1'][f_1'] Y_1' I_1' J_1' M_{I_1}' M_{J_1}' \right\rangle = \delta_{11'} \tag{15}$$

here $\delta_{11'}$ is a product of the $\delta_{\nu_1 \nu_1'}, \delta_{mm'}, \cdots$, which includes every pair of quantum numbers. The only complication is caused by the non-orthogonality of the single quark orbital state,

$$\left\langle \begin{smallmatrix} [\nu_1]W_1^x \\ m \end{smallmatrix} \middle| \begin{smallmatrix} [\nu_1']W_1'^x \\ m' \end{smallmatrix} \right\rangle = \delta_{\nu_1 \nu_1'} \delta_{mm'} \left\langle \begin{smallmatrix} [\nu_1]W_1^x \\ m \end{smallmatrix} \middle| \begin{smallmatrix} [\nu_1']W_1'^x \\ m' \end{smallmatrix} \right\rangle \tag{16}$$

Finally we have the four quark overlap

$$\begin{aligned} \langle \alpha_1 K_1 | \alpha_1' K_1' \rangle &= \delta_{11'} h_{\nu_1}^{-1} \sum_m \left\langle \begin{smallmatrix} [\nu_1]W_1^x \\ m \end{smallmatrix} \middle| \begin{smallmatrix} [\nu_1]W_1'^x \\ m \end{smallmatrix} \right\rangle \\ &= \delta_{11'} \left\langle \begin{smallmatrix} [\nu_1]W_1^x \\ m \end{smallmatrix} \middle| \begin{smallmatrix} [\nu_1]W_1'^x \\ m \end{smallmatrix} \right\rangle \text{(any m)} \end{aligned} \tag{17}$$

This four-body overlap is listed by Harvey in his table 6[9]. To be consistent with the standard $SU^x(2)$ CGC phase convention, all the entries in his table 6 should have positive signs. Another modification is caused by the delocalized orbit eq.(4): The m in Harvey's table 6 should be replaced by

$$m \to (2\epsilon + (1+\epsilon^2)F)/(1+\epsilon^2+2\epsilon F), F = \langle \phi_L | \phi_R \rangle .$$

Harvey's result is our $\epsilon = 0$ limit.

The two-quark state

$$|\ \alpha_2 K_2\ \rangle = \left| [\sigma_2] W_2^c [\mu_2][f_2] \overset{[\nu_2]W_2^x}{Y_2} I_2 J_2 M_{I_2} M_{J_2} \right\rangle \tag{18}$$

can be expressed in a similar form as eq.(14). But the $[\nu_2], [\sigma_2], [\mu_2]$, and$[\mathrm{f}_2]$ are either symmetric [2] or antisymmetric$[1^2]$, and eq.(18) is in fact just a product of orbital, color, flavor, and spin part. The two-body interaction matrix elements can be factorized too,

$$\langle \alpha_2 K_2 | H_{56} | \alpha_2' K_2' \rangle = \left\langle \begin{smallmatrix} [\nu_2] \\ W_2^x \end{smallmatrix} \middle| H_{56}^x \middle| \begin{smallmatrix} [\nu_2'] \\ W_2'^x \end{smallmatrix} \right\rangle \left\langle \begin{smallmatrix} [\sigma_2] \\ W_2^c \end{smallmatrix} \middle| H_{56}^c \middle| \begin{smallmatrix} [\sigma_2' \\ W'^x_{W_2'^c} \end{smallmatrix} \right\rangle$$

$$\left\langle \begin{smallmatrix} [f_2] \\ Y_2 I_2 M_{I_2} \end{smallmatrix} \middle| H_{56}^f \middle| \begin{smallmatrix} [f_2'] \\ Y_2' I_2' M_{I_2}' \end{smallmatrix} \right\rangle \langle J_2 M_{J_2} | H_{56}^\sigma | J_2' M_{J_2}' \rangle. \tag{19}$$

Here we have used the fact that the two body interaction is a sum of terms of the form which we take as a single term for simplicity below.

$$H_{56} = H_{56}^c \cdot H_{56}^f \cdot H_{56}^\sigma \cdot H_{56}^x \tag{20}$$

For the nonrelativistic case, H is a scalar of $SU^c(3), SU^\tau(2)$ and $SU^\sigma(2)$, the two-body matrix elements are W_2^c, M_{I_2} and M_{J_2} independent, and the first three CGC in eq.(13) will disappear in the matrix element $\langle \Phi_{\alpha K} | H | \Phi_{\alpha K'} \rangle$ of eq.(12) due to the orthonormal property of CGC's.

For the one body operator (kinetic energy in a nonrelativistic model, kinetic energy and mean field in a relativistic model)

$$H_{56} = H_5 + H_6,$$

by expanding the coupled state into the product of two particle states with CGC and using the orthonormal property of CGC, the two one-body operator matrix elements can be calculated very easily. The $6 \to 5 + 1fp$ expansion can be avoided and only the $6 \to 4 + 2fp$ coefficients need to be included in a computer program package.

IV THE RELATIVISTIC EXTENSION

It is commonly believed that the classification scheme eq.(2) can be applied to the nonrelativistic quark only, because the spin and orbital part are intrinsically coupled into a Dirac spinor for a relativistic quark. However in a Dirac cluster model, only the lowest Dirac state is used and the lowest state of a Dirac particle moving in a central potential can be expressed as a product of a pseudo-orbit and a Pauli spinor[14]

$$\phi_\sigma(\mathbf{r}') = \begin{pmatrix} \phi_u(\mathbf{r}') \\ -i\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}' \phi_{\alpha'}(\mathbf{r}') \end{pmatrix} \chi_\sigma Y_{00}(\theta', \phi'). \tag{21}$$

Here $\mathbf{r}' = \mathbf{r} - \mathbf{s}/2$ or $\mathbf{r} + \mathbf{s}/2$ depends on the confinement center, χ_σ is the usual Pauli spinor, $\sigma = j_z = \pm 1/2, Y_{00} = \sqrt{\frac{1}{4\pi}}$, ϕ_u and ϕ_d are the upper and lower (down)

components of the Dirac WF. Taking the $\sqrt{\frac{1}{4\pi}}\binom{\phi_u(\mathbf{r}')}{-i\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}'\phi_d(\mathbf{r}')}$ as a pseudo-orbit WF equivalent to that for the nonrelativistic orbital WF, we obtain two linear independent states as the bases of a pseudo-orbit $SU^x(2)$ for the Dirac quark. In this way we can use the same classification scheme eq.(2) to describe the six Dirac quark system[15]. The whole calculation method discussed in Sec. II and III can be extended to a Dirac quark cluster model directly. The only difference is that when we calculate the one- and two-body matrix elements, we have to recombine the pseudo-orbit and the Pauli spinor together to be a Dirac spinor. For the four quark overlap calculation, recombination of the pseudo-orbit and Pauli spinor seems to be needed too. However, because we only use the lowest Dirac state WF eq.(21), the single particle overlap still can be separated into a pseudo-orbit part and a Pauli spinor part

$$
\begin{aligned}
< \psi_{\sigma_1}(\mathbf{r}_1)|\psi_{\sigma_2}(\mathbf{r}_2) &= \chi^\dagger_{\sigma_1}\frac{1}{4\pi}\int d\mathbf{r}(\psi_u(\mathbf{r}_1), i\boldsymbol{\sigma}\cdot\mathbf{r}_1\psi_d(\mathbf{r}_1))(\psi_u(\mathbf{r}_2), -i\boldsymbol{\sigma}\cdot\mathbf{r}_2\psi_d(\mathbf{r}_2))\chi_{\sigma_2} \\
&= \chi^\dagger_{\sigma_1}\frac{1}{4\pi}\int d\mathbf{r}[\psi_u(\mathbf{r}_1)\psi_u(\mathbf{r}_2) + \boldsymbol{\sigma}\cdot\mathbf{r}_1\boldsymbol{\sigma}\cdot\mathbf{r}_2\psi_d(\mathbf{r}_1)\psi_d(\mathbf{r}_2)]\chi_{\sigma_2} \\
&= \chi^\dagger_{\sigma_1}\chi_{\sigma_2}\frac{1}{4\pi}\int d\mathbf{r}[\psi_u(\mathbf{r}_1)\psi_u(\mathbf{r}_2) + \hat{\mathbf{r}}_1\cdot\hat{\mathbf{r}}_2\psi_d(\mathbf{r}_1)\psi_d(\mathbf{r}_2)]. \qquad (22)
\end{aligned}
$$

The spin dependent part is identically zero[16],

$$
\frac{1}{4\pi}\int i\boldsymbol{\sigma}\cdot(\hat{\mathbf{r}}_1\times\hat{\mathbf{r}}_2)\psi_d(\mathbf{r}_1)\psi_d(\mathbf{r}_2)d\mathbf{r} \equiv 0. \qquad (23)
$$

Therefore the four quark overlap calculation can be done in an exactly same way as that for the nonrelativistic case, i.e., separated into a pseudo-orbital part and a spin part.

V. COMPUTERIZED FRACTIONAL PARENTAGE EXPANSION

All the needed $SU(mn) \supset SU(m)\times SU(n)$ isoscalar factors can be found directly from Chen's book[8b], the needed $SU(3) \supset SU(2)\times U(1)$ isoscalar factors can be obtained from Chen's $SU(3)$ CGC[8b] and the standard $SU(2)$ CGC. (Some $SU(3)$ CGC not explicitly listed there can be obtained by the symmetry properties from the listed ones. Table 2 gives the additional needed phase factors ϵ_2 which are missing in table 5 of Sec. II of ref.[8b].)

It is time consuming and requires a good grasp of group theory to combine the individual isoscalar factors into the transformation coefficients between physical bases and symmetry bases and the $6 \rightarrow 4+2$ cfp for the matrix element calculations.

In order to make the calculation automatic and to facilitate others using this fp expansion technique, a computer program has been written. All the needed isoscalar factors are stored in the program. After inputting the quantum numbers $\alpha = (YIJ)$, the program will automatically yield the physical bases, symmetry bases, the transformation coefficients between these two bases and the $6 \rightarrow 4+2$ cfp for the symmetry bases. This part may be useful for other dibaryon model practitioners if they want to use the fp expansion methods. For our own problem, the program continues on to calculate the one body, two body matrix elements, the four body overlap, combine them together into the six quark Hamiltonian matrix elements in the physical bases, diagonalize the Hamiltonian in the non-orthogonal physical basis space, minimize the eigen-energy and fix the eigen-WF with respect to the delocalization parameter $\epsilon(s)$, and repeat this calculation for different separations s between two q^3 clusters from $s =$ 0.1 to 3 fm, and finally outputs the adiabatic potential $V_\alpha(s)$. This program greatly reduced the labor involved in the systematic search of dibaryon candidates in the u, d and s three flavor world. Only minor modification of the subroutine for the one and two body matrix elements calculation, suffices to adapt the program to a relativistic quark model dibaryon search. We expect it is also easy to apply this program to other nonrelativistic and relativistic dibaryon calculations with minor modifications, particularly as the fp expansion part is universal for this kind of dibaryon model calculations.

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APPENDIX

Our Tables for the symmetry decompositions might appear to contradict Harvey's results [9]. This is due to a difference in terminology: Harvey used 'symmetric' and 'antisymmetric' to refer only to the <u>orbital</u> components when discussing non-identical particles. We prefer to use the more inclusive definition below, since we believe it allows for a more natural relation to the identical particle case.

The symmetric (antisymmetric) combination $\bar{xy}(\widetilde{xy})$ of two baryon state is defined as

$$\bar{xy} = \frac{xy+yx}{\sqrt{2}}, \qquad \widetilde{xy} = \frac{xy-yx}{\sqrt{2}} \tag{A1}$$

Let's use the $N\Delta$ two baryon state as an example to show the symmetry property. Below χ_c is the color singlet three quark state, $N_{m_N\tau_N}(\Delta_{m_\Delta\tau_\Delta})$ is a three quark $N(\Delta)$ spin-isospin symmetric state with spin-isospin projection quantum numbers $m_N\tau_N(m_\Delta\tau_\Delta)$, $l(123)$ is a product orbital state $l(1)l(2)l(3)$ and l is defined in eq. (4), $r(456)$ has the parallel meaning, $C^{Kk}_{K_ak_a,K_bk_b}$ is the spin (isospin) CGC. Then

$$\begin{aligned}
(N\Delta)_{IJ} &= A\Sigma C^{Jm}_{J_Nm_N,J_\Delta m_\Delta} C^{I\tau}_{I_N\tau_N,I_\Delta\tau_\Delta} \\
&\chi_c(123)N_{m_N\tau_N}(123)l(123)\chi_c(456)\Delta_{m_\Delta\tau_\Delta}(456)r(456) \\
&= \frac{1}{\sqrt{20}}\Sigma C^{Jm}_{J_Nm_N,J_\Delta m_\Delta} C^{I\tau}_{I_N\tau_N,I_\Delta\tau_\Delta}\{\chi_c(123)\chi_c(456) \\
&[N_{m_N\tau_N}(123)\Delta_{m_\Delta\tau_\Delta}(456)l(123)r(456) \\
&-N_{m_N\tau_N}(456)\Delta_{m_\Delta\tau_\Delta}(123)l(456)r(123)] + ...\}
\end{aligned} \tag{A2}$$

$$\begin{aligned}
(\Delta N)_{IJ} &= A\Sigma C^{Jm}_{J_\Delta m_\Delta,J_Nm_N} C^{I\tau}_{I_\Delta\tau_\Delta,I_N\tau_N} \\
&\chi_c(123)\Delta_{m_\Delta\tau_\Delta}(123)l(123)\chi_c(456)N_{m_N\tau_N}(456)r(456) \\
&= \frac{1}{\sqrt{20}}\Sigma C^{Jm}_{J_\Delta m_\Delta,J_Nm_N} C^{I_\tau}_{I_\Delta\tau_\Delta,I_N\tau_N}\{\chi_c(123)\chi_c(456) \\
&[\Delta_{m_\Delta\tau_\Delta}(123)N_{m_N\tau_N}(456)l(123)r(456) \\
&-\Delta_{m_\Delta\tau_\Delta}(456)N_{m_N\tau_N}(123)l(456)r(123)] + ...\}
\end{aligned} \tag{A.3}$$

where +... represents all of the other permutations.

$$\begin{aligned}
(\overline{N\Delta})_{IJ} &= \frac{(N\Delta)_{IJ}+(\Delta N)_{IJ}}{\sqrt{2}} \\
&= \frac{1}{\sqrt{40}}\Sigma C^{Jm}_{J_Nm_N,J_\Delta m_\Delta} C^{I\tau}_{I_N\tau_N,I_\Delta\tau_\Delta}\{\chi_c(123)\chi_c(456)
\end{aligned}$$

$$\Big[N_{m_N\tau_N}(123)\Delta_{m_\Delta\tau_\Delta}(456)\left(l(123)r(456)-(-)^{J_N+J_\Delta-J+I_N+I_\Delta-I}l(456)r(123)\right)$$
$$-N_{m_N\tau_N}(456)\Delta_{m_\Delta\tau_\Delta}(123)\left(l(456)r(123)-(-)^{J_N+J_\Delta-J+I_N+I_\Delta-I}l(123)r(456)\right)\Big]+....\} \tag{A.4}$$

The orbital symmetry property (the parity) of a two baryon state under the permutation $\binom{123,456}{456,123}$ is dependent on the spin-isospin quantum numbers, instead of directly related to the symmetry (antisymmetry) $\bar{x}y(\widetilde{xy})$ combination as explained in [9].

In deriving (A.4), we have used the well known $SU(2)$ relation

$$C^{K_k}_{K_ak_aK_bk_b}=(-)^{K_a+K_b-K}C^{Kk}_{K_bk_bK_ak_a} \tag{A.5}$$

Note, for example, that if the Δ were replaced by a second N, and $I+J$ is even, the first and fourth terms become identical as do the second and third terms, etc. The result has only antisymmetric orbital parts. The $(\overline{N}N)_{IJ}\equiv 0$ for odd $I+J$. Conversely, for $\overline{(\overline{N}N)}_{IJ}$, only the odd $I+J$ symmetric orbital parts exist, (as for example, in the deuteron).

TABLE CAPTIONS

1. TABLE 1a. The transformation coefficients between physical bases and symmetry bases for Y = 2. The head of columns is $[\nu]$, $[\mu]$, $[f]$: 1 - [6]; 2 - [51]; 3 - [42]; 4 - [33]; 5 - [411]; 6 - [321]; 7 - [222]. The head of rows is B_1B_2: 1 - N; 2 -Σ; 3 - Ξ; 4 - Λ; 5 - Δ; 6 - Σ^*; 7 - Ξ^*; 8 - Ω. $\bar{xy}(\widetilde{xy})$ means symmetric (antisymmetric) channel of baryons x and y. The transformation coefficients should be the square root of the entries, and a negative sign means to take the negative square root.
2. TABLE 1b. Same as table 1a. for Y = 1.
3. TABLE 1c. Same as table 1a. for Y = 0.
4. TABLE 1d. Same as table 1a. for Y = -1.
5. TABLE 1e. Same as table 1a. for Y = -2.
6. TABLE 1f. Same as table 1a. for Y = -3.
7. TABLE 1g. Same as table 1a. for Y = -4.
8. TABLE 2. The additional phase factor $\epsilon_2(\nu_1\nu_2\nu)$.

TABLE 1a

IJ = 33	411					
55	1					
IJ = 32	321					
55	1					
IJ = 31	231	431				
55	-5/9	-4/9				
IJ = 30	141	341				
55	-1/5	-4/5				
IJ = 23	322					
55	1					
IJ = 22	232	322	412	432		
$\overline{15}$	1/9	0	4/5	4/45		
$\widetilde{51}$	0	-1	0	0		
55	-4/9	0	1/5	-16/45		
IJ = 21	142	232	322	342	432	
$\overline{15}$	4/45	0	5/9	16/45	0	
$\widetilde{15}$	0	5/9	0	0	4/9	
55	-1/9	0	4/9	-4/9	0	
IJ = 20	232	432				
55	-5/9	-4/9				
IJ = 13	233	433				
55	-5/9	-4/9				
IJ = 12	143	233	323	343	4133	
$\overline{15}$	4/45	0	5/9	16/45	0	
$\widetilde{15}$	0	5/9	0	0	4/9	
55	-1/9	0	4/9	-4/9	0	
IJ = 11	$2_1 33$	$2_2 33$	323	413	$4_1 33$	$4_2 33$
11	5/81	20/81	0	4/9	4/81	16/81
$\overline{15}$	-20/81	-5/81	0	4/9	-16/81	-4/81
$\widetilde{15}$	0	0	-1	0	0	0
55	20/81	-20/81	0	1/9	16/81	-16/81
IJ = 10	143	323	343			
11	1/9	4/9	4/9			
55	-4/45	5/9	-16/45			
IJ = 03	144	344				
55	-1/5	-4/5				
IJ = 02	234	434				
55	-5/9	-4/9				
IJ = 01	144	324	344			
11	1/9	4/9	4/9			
55	-4/45	5/9	-16/45			
IJ = 00	234	414	434			
11	1/9	4/5	4/45			
55	-4/9	1/5	-16/45			

TABLE 1b

IJ=$\frac{5}{2}$3	322	411
$\overline{56}$	0	1
$\widetilde{56}$	1	0

IJ=$\frac{5}{2}$2	232	321	322	412	432
$\overline{25}$	1/9	0	0	4/5	4/45
$\widetilde{25}$	0	0	-1	0	0
$\overline{56}$	0	1	0	0	0
$\widetilde{56}$	-4/9	0	0	1/5	-16/45

IJ=$\frac{5}{2}$1	142	231	232	322	342	431	432
$\overline{25}$	4/45	0	0	5/9	16/45	0	0
$\widetilde{25}$	0	0	5/9	0	0	0	4/9
$\overline{56}$	0	-5/9	0	0	0	-4/9	0
$\widetilde{56}$	-1/9	0	0	4/9	-4/9	0	0

IJ=$\frac{3}{2}$0	141	232	341	432
$\overline{56}$	-1/5	0	-4/5	0
$\widetilde{56}$	0	-5/9	0	-4/9

IJ=$\frac{3}{2}$3	233	322	433
$\overline{56}$	0	1	0
$\widetilde{56}$	-5/9	0	-4/9

IJ=$\frac{3}{2}$2	143	232	233	235	322	323	325	343	412	432
$\overline{16}$	0	5/72	5/72	-5/36	0	0	0	0	1/2	1/18
$\overline{25}$	1/36	0	0	0	-1/16	25/144	-5/8	1/9	0	0
$\overline{45}$	0	5/144	-5/16	5/72	0	0	0	0	1/4	1/36
$\widetilde{25}$	0	1/144	25/144	25/72	0	0	0	0	1/20	1/180
$\widetilde{15}$	-1/20	0	0	0	-5/16	-5/16	-1/8	-1/5	0	0
$\overline{56}$	0	-4/9	0	0	0	0	0	0	1/5	-16/45
$\widetilde{16}$	1/90	0	0	0	-5/8	5/72	1/4	2/45	0	0
$\widetilde{56}$	-1/9	0	0	0	0	4/9	0	-4/9	0	0

	433	435
$\overline{16}$	1/18	-1/9
$\overline{25}$	0	0
$\overline{45}$	-1/4	1/18
$\widetilde{25}$	5/36	5/18
$\widetilde{45}$	0	0
$\overline{56}$	0	0
$\widetilde{16}$	0	0
$\widetilde{56}$	0	0

TABLE 1b Cont-

IJ=$\frac{3}{2}$1	142	145	232	23_13	23_23	235	322	323	325	342
$\overline{16}$	1/18	-2/45	0	0	0	0	25/72	-1/8	1/36	2/9
$\overline{25}$	0	0	5/144	-25/324	-25/1296	25/72	0	0	0	0
$\overline{45}$	1/36	1/45	0	0	0	0	25/144	9/16	-1/72	1/9
$\widetilde{25}$	1/180	1/9	0	0	0	0	5/144	-5/16	-5/72	1/45
$\widetilde{45}$	0	0	25/144	5/36	5/144	5/72	0	0	0	0
$\overline{56}$	-1/9	0	0	0	0	0	4/9	0	0	-4/9
$\widetilde{16}$	0	0	25/72	-5/162	-5/648	-5/36	0	0	0	0
$\widetilde{56}$	0	0	0	20/81	-20/81	0	0	0	0	0
$\overline{12}$	0	0	0	5/81	20/81	0	0	0	0	0
$\widetilde{12}$	0	1/45	0	0	0	0	0	0	8/9	0

	345	413	432	43_13	43_23	435
$\overline{16}$	-3/45	0	0	0	0	0
$\overline{25}$	0	5/36	1/36	-5/81	-5/324	5/18
$\overline{45}$	4/45	0	0	0	0	0
$\widetilde{25}$	4/9	0	0	0	0	0
$\widetilde{45}$	0	-1/4	5/36	1/9	1/36	1/18
$\overline{56}$	0	0	0	0	0	0
$\widetilde{16}$	0	1/18	5/18	-2/81	-1/162	-1/9
$\widetilde{56}$	0	1/9	0	16/81	-16/81	0
$\overline{12}$	0	4/9	0	4/81	16/81	0
$\widetilde{12}$	4/45	0	0	0	0	0

IJ=$\frac{3}{2}$0	143	232	235	323	343	432	435
$\overline{12}$	1/9	0	0	4/9	4/9	0	0
$\widetilde{12}$	0	0	-5/9	0	0	0	-4/9
$\overline{56}$	0	-5/9	0	0	0	-4/9	0
$\widetilde{56}$	-4/45	0	0	5/9	-16/45	0	0

IJ=$\frac{1}{2}$3	144	233	344	433
$\overline{56}$	0	-5/9	0	-4/9
$\widetilde{56}$	-1/5	0	-4/5	0

IJ=$\frac{1}{2}$2	143	146	233	234	236	323	343	346	433	434	436
$\overline{16}$	16/255	-1/25	0	0	0	4/9	64/225	-4/25	0	0	0
$\overline{25}$	0	0	1/9	0	4/9	0	0	0	4/45	0	16/45
$\widetilde{25}$	4/225	4/25	0	0	0	1/9	16/225	16/25	0	0	0
$\overline{56}$	-1/9	0	0	0	0	4/9	-4/9	0	0	0	0
$\widetilde{16}$	0	0	4/9	0	-1/9	0	0	0	16/45	0	-4/45
$\widetilde{56}$	0	0	0	-5/9	0	0	0	0	0	-4/9	0

IJ=$\frac{1}{2}$1	144	146	23_13	23_23	23_16	23_26	323	324	326	344
$\overline{12}$	1/18	2/45	0	0	0	0	0	2/9	-5/18	2/9
$\overline{14}$	0	0	1/18	2/9	-1/36	-1/36	0	0	0	0
$\widetilde{12}$	0	0	-1/162	-2/81	-1/4	-1/4	0	0	0	0
$\widetilde{14}$	1/18	-2/25	0	0	0	0	0	2/9	5/18	2/9
$\overline{56}$	0	0	20/81	-20/81	0	0	0	0	0	0
$\widetilde{56}$	-4/45	0	0	0	0	0	0	5/9	0	-16/45
$\overline{16}$	0	0	-16/81	-4/81	-1/18	1/18	0	0	0	0
$\overline{25}$	0	-4/45	0	0	0	0	-1/5	0	-16/45	0
$\widetilde{25}$	0	0	-4/81	-1/81	2/9	-2/9	0	0	0	0
$\widetilde{16}$	0	1/45	0	0	0	0	-4/5	0	4/45	0

TABLE 1b Cont-

	346	413	43_13	43_23	43_16	43_26			
$\overline{12}$	8/45	0	0	0	0	0			
$\overline{14}$	0	2/5	2/45	8/45	-1/45	-1/45			
$\widetilde{12}$	0	-2/45	-2/405	-8/405	-1/5	-1/5			
$\widetilde{14}$	-8/45	0	0	0	0	0			
$\overline{56}$	0	1/9	16/81	-16/81	0	0			
$\widetilde{56}$	0	0	0	0	0	0			
$\overline{16}$	0	16/45	-64/405	-16/405	-2/45	2/45			
$\overline{25}$	-16/45	0	0	0	0	0			
$\widetilde{25}$	0	4/45	-16/405	-4/405	8/45	-8/45			
$\widetilde{16}$	4/45	0	0	0	0	0			
IJ=$\frac{1}{2}$0	143	234	236	323	326	343	414	434	436
$\overline{12}$	0	1/18	5/18	0	0	0	2/5	2/45	2/9
$\overline{14}$	1/10	0	0	2/5	1/10	2/5	0	0	0
$\widetilde{12}$	-1/90	0	0	-2/45	9/10	-2/45	0	0	0
$\widetilde{14}$	0	1/18	-5/18	0	0	0	2/5	2/45	-2/9
$\overline{56}$	-4/45	0	0	5/9	0	-16/45	0	0	0
$\widetilde{56}$	0	-4/9	0	0	0	0	1/5	-16/45	0

TABLE 1c

IJ=23	233	322	411	433
$\overline{57}$	-1/3	0	2/5	-4/15
66	2/9	0	3/5	8/45
$\widetilde{57}$	0	1	0	0

IJ=22	143	232	233	321	322	323	343	412	432	433
$\overline{26}$	0	1/12	5/36	0	0	0	0	3/5	1/15	1/9
$\overline{35}$	0	1/36	-5/12	0	0	0	0	1/5	1/45	-1/3
$\widetilde{35}$	-1/15	0	0	0	-1/4	-5/12	-4/15	0	0	0
$\overline{57}$	-1/15	0	0	2/5	0	4/15	-4/15	0	0	0
$\widetilde{26}$	1/45	0	0	0	-3/4	5/36	4/45	0	0	0
66	2/45	0	0	3/5	0	-8/45	8/45	0	0	0
$\widetilde{57}$	0	-4/9	0	0	0	0	0	1/5	-16/45	0

IJ=21	142	231	232	23_13	23_23	322	323	342	413	431
$\overline{26}$	1/15	0	0	0	0	5/12	-1/4	4/15	0	0
$\overline{35}$	1/45	0	0	0	0	5/36	3/4	4/45	0	0
$\widetilde{35}$	0	0	5/36	5/27	5/108	0	0	0	-1/3	0
$\overline{57}$	0	-2/9	0	4/27	-4/27	0	0	0	1/15	-8/45
$\widetilde{26}$	0	0	5/12	-5/81	-5/324	0	0	0	1/9	0
$\widetilde{57}$	-1/9	0	0	0	0	4/9	0	-4/9	0	0
66	0	-1/3	0	-8/81	8/81	0	0	0	-2/45	-4/15
22	0	0	0	5/81	20/81	0	0	0	4/9	0

	432	43_13	43_23
$\overline{26}$	0	0	0
$\overline{35}$	0	0	0
$\widetilde{35}$	1/9	4/27	1/27
$\overline{57}$	0	16/135	-16/135
$\widetilde{26}$	1/3	-4/81	-1/81
$\widetilde{57}$	0	0	0
66	0	-32/405	32/405
22	0	4/31	16/31

IJ=20	141	143	232	323	341	343	432
$\overline{57}$	-2/25	-4/75	0	1/3	-8/25	-16/75	0
66	-3/25	8/225	0	-2/9	-12/25	32/225	0
$\widetilde{57}$	0	0	-5/9	0	0	0	-4/9
22	0	1/9	0	4/9	0	4/9	0

TABLE 1c Cont-

IJ=13	144	233	322	344	433
$\overline{57}$	-2/15	0	1/3	-3/15	0
66	1/15	0	2/3	4/15	0
$\widetilde{57}$	0	5/9	0	0	-4/9

IJ=12	143	146	232	233	234	235	236	322	323	325
$\overline{17}$	0	0	1/27	1/9	0	-5/27	-2/27	0	0	0
$\overline{26}$	1/25	2/75	0	0	0	0	0	-1/12	1/4	-1/3
$\overline{35}$	-1/225	-8/75	0	0	0	0	0	-1/12	-1/36	-1/3
$\overline{46}$	0	0	1/18	-1/6	0	0	1/9	0	0	0
$\widetilde{35}$	0	0	1/108	-1/36	0	5/27	-8/27	0	0	0
$\overline{57}$	0	0	-4/27	0	-10/27	0	0	0	0	0
$\widetilde{26}$	0	0	1/108	1/4	0	5/27	2/27	0	0	0
$\widetilde{46}$	-2/75	1/25	0	0	0	0	0	-1/2	-1/6	0
$\widetilde{17}$	4/225	-2/75	0	0	0	0	0	-1/3	1/9	1/3
$\widetilde{57}$	-1/9	0	0	0	0	0	0	0	4/9	0
66	0	0	-8/27	0	5/27	0	0	0	0	0

	343	346	412	432	433	434	435	436
$\overline{17}$	0	0	4/15	4/135	4/45	0	-4/27	-8/135
$\overline{26}$	4/25	8/75	0	0	0	0	0	0
$\overline{35}$	-4/225	-32/75	0	0	0	0	0	0
$\overline{46}$	0	0	2/5	2/45	-2/15	0	0	4/45
$\widetilde{35}$	0	0	1/15	1/135	-1/45	0	4/27	-32/135
$\overline{57}$	0	0	1/15	-16/135	0	-8/27	0	0
$\widetilde{26}$	0	0	1/15	1/135	1/5	0	4/27	8/135
$\widetilde{46}$	-8/75	4/25	0	0	0	0	0	0
$\widetilde{17}$	16/225	-8/75	0	0	0	0	0	0
$\widetilde{57}$	-4/9	0	0	0	0	0	0	0
66	0	0	2/15	-32/135	0	4/27	0	0

IJ=11	142	144	145	146	232	23_13	23_23	235	23_16	23_26
$\overline{17}$	4/135	0	-8/135	2/135	0	0	0	0	0	0
$\overline{26}$	0	0	0	0	5/108	-1/9	-1/36	5/27	1/27	-1/27
$\overline{35}$	0	0	0	0	5/108	1/81	1/324	5/27	-4/27	4/27
$\overline{46}$	2/45	0	0	-1/45	0	0	0	0	0	0
$\widetilde{35}$	1/135	0	8/135	8/135	0	0	0	0	0	0
$\overline{57}$	-1/27	-8/135	0	0	0	0	0	0	0	0
$\widetilde{26}$	1/135	0	8/135	-2/135	0	0	0	0	0	0
$\widetilde{46}$	0	0	0	0	5/18	2/27	1/54	0	1/18	-1/18
66	-2/27	4/135	0	0	0	0	0	0	0	0

TABLE 1c Cont-

$\widetilde{17}$	0	0	0	0	5/27	-4/81	-1/81	-5/27	-1/27	1/27
$\widetilde{57}$	0	0	0	0	0	20/81	-20/81	0	0	0
$\overline{13}$	0	0	0	0	0	2/81	8/81	0	-1/6	-1/6
22	0	1/54	1/270	8/135	0	0	0	0	0	0
$\overline{24}$	0	0	0	0	0	1/27	4/27	0	1/9	1/9
$\widetilde{13}$	0	1/27	1/135	-4/135	0	0	0	0	0	0
$\widetilde{42}$	0	1/18	-1/90	0	0	0	0	0	0	0

	322	323	324	325	326	342	344	345	346	413
$\overline{17}$	5/27	-1/5	0	1/27	8/135	16/135	0	-32/135	8/135	0
$\overline{26}$	0	0	0	0	0	0	0	0	0	1/5
$\overline{35}$	0	0	0	0	0	0	0	0	0	-1/45
$\overline{46}$	5/18	3/10	0	0	-4/45	8/45	0	0	-4/45	0
$\widetilde{35}$	5/108	1/20	0	-1/27	32/135	4/135	0	32/135	32/135	0
$\overline{57}$	4/27	0	10/27	0	0	-4/27	-32/135	0	0	0
$\widetilde{26}$	5/108	-9/20	0	-1/27	-8/135	24/135	0	32/135	-8/135	0
$\widetilde{46}$	0	0	0	0	0	0	0	0	0	-2/15
66	8/27	0	-5/27	0	0	-8/27	16/135	0	0	0
$\widetilde{17}$	0	0	0	0	0	0	0	0	0	4/45
$\widetilde{57}$	0	0	0	0	0	0	0	0	0	1/9
$\overline{13}$	0	0	0	0	0	0	0	0	0	8/45
22	0	0	2/27	4/27	-10/27	0	2/27	2/135	32/135	0
$\overline{24}$	0	0	0	0	0	0	0	0	0	4/15
$\widetilde{13}$	0	0	4/27	8/27	5/27	0	4/27	4/135	-16/135	0
$\widetilde{42}$	0	0	2/9	-4/9	0	0	2/9	-2/45	0	0

	432	43_13	43_23	435	43_16	43_26
$\overline{17}$	0	0	0	0	0	0
$\overline{26}$	1/27	-4/45	-1/45	4/27	4/135	-4/135
$\overline{35}$	1/27	4/405	1/405	4/27	-16/135	16/135
$\overline{46}$	0	0	0	0	0	0
$\widetilde{35}$	0	0	0	0	0	0
$\overline{57}$	0	0	0	0	0	0
$\widetilde{26}$	0	0	0	0	0	0
$\widetilde{46}$	2/9	8/135	2/135	0	2/45	-2/45
66	0	0	0	0	0	0
$\widetilde{17}$	4/27	-16/405	-4/405	-4/27	-4/135	4/135
$\widetilde{57}$	0	16/81	-16/81	0	0	0
$\overline{13}$	0	8/405	32/405	0	-2/15	-2/15
22	0	0	0	0	0	0
$\overline{24}$	0	4/135	16/135	0	4/45	4/45
$\widetilde{13}$	0	0	0	0	0	0
$\widetilde{42}$	0	0	0	0	0	0

TABLE 1c Cont-

IJ=10	143	232	234	235	236	323	326	343	414	432
$\overline{13}$	2/45	0	0	0	0	8/45	3/5	8/45	0	0
$\overline{24}$	1/15	0	0	0	0	4/15	-2/5	4/15	0	0
$\widetilde{13}$	0	0	1/27	-5/27	-5/27	0	0	0	4/15	0
$\widetilde{42}$	0	0	1/18	5/18	0	0	0	0	2/5	0
$\overline{57}$	0	-5/27	-8/27	0	0	0	0	0	2/15	-4/27
$\widetilde{57}$	-4/45	0	0	0	0	5/9	0	-16/45	0	0
66	0	-10/27	4/27	0	0	0	0	0	-1/15	-8/27
22	0	0	1/54	-5/54	10/27	0	0	0	2/15	0

	434	435	436
$\overline{13}$	0	0	0
$\overline{24}$	0	0	0
$\widetilde{13}$	4/135	-4/27	-4/27
$\widetilde{42}$	2/45	2/9	0
$\overline{57}$	-32/135	0	0
$\widetilde{57}$	0	0	0
66	16/135	0	0
22	2/135	-2/27	8/27

IJ=03	233	433
66	-5/9	-4/9

IJ=02	143	146	233	236	323	343	346	433	436
$\overline{17}$	4/75	-2/25	0	0	1/3	16/75	-8/25	0	0
$\overline{26}$	0	0	2/9	1/3	0	0	0	8/45	4/15
$\widetilde{26}$	8/225	3/25	0	0	2/9	32/225	12/25	0	0
66	-1/9	0	0	0	4/9	-4/9	0	0	0
$\widetilde{17}$	0	0	1/3	-2/9	0	0	0	4/15	-8/45

IJ=01	146	23_13	23_23	23_16	23_26	237	323	326	346	413
$\overline{13}$	4/45	0	0	0	0	0	0	-5/9	16/45	0
$\overline{17}$	0	-4/27	-1/27	-1/9	1/9	0	0	0	0	4/15
$\overline{26}$	-1/15	0	0	0	0	0	-2/5	-4/15	-4/15	0
$\widetilde{13}$	0	-1/54	-2/27	-1/18	-1/18	5/18	0	0	0	-2/15
$\widetilde{26}$	0	-8/81	-2/81	1/6	-1/6	0	0	0	0	8/45
$\widetilde{17}$	2/45	0	0	0	0	0	-3/5	8/45	8/45	0
22	0	-1/648	-1/162	-1/6	-1/6	-5/24	0	0	0	-1/90
44	0	1/24	1/6	-1/18	-1/18	5/72	0	0	0	3/10
66	0	20/81	-20/81	0	0	0	0	0	0	1/9

TABLE 1c Cont-

	$43_1 3$	$43_2 3$	$43_1 6$	$43_2 6$	437			
$\overline{13}$	0	0	0	0	0			
$\overline{17}$	-16/135	-4/135	-4/45	4/45	0			
$\overline{26}$	0	0	0	0	0			
$\widetilde{13}$	-2/135	-8/135	-2/45	-2/45	2/9			
$\widetilde{26}$	-32/405	-8/405	2/15	-2/15	0			
$\widetilde{17}$	0	0	0	0	0			
22	-1/810	-2/405	-2/15	-2/15	-1/6			
44	1/30	2/15	-2/45	-2/45	1/18			
66	16/81	-16/81	0	0	0			
IJ=00	143	147	236	323	326	343	347	436
$\overline{13}$	0	0	5/9	0	0	0	0	4/9
22	-1/360	3/40	0	-1/90	3/5	-1/90	3/10	0
$\widetilde{13}$	-1/30	-1/10	0	-2/15	1/5	-2/15	-2/5	0
44	3/40	-1/40	0	3/10	1/5	3/10	-1/10	0
66	-4/45	0	0	5/9	0	-16/45	0	0

TABLE 1d

IJ=$\frac{3}{2}$3	144	233	322	344	411	433
$\overline{58}$	0	-1/2	0	0	1/10	-2/5
$\overline{67}$	0	1/18	0	0	9/10	2/45
$\widetilde{67}$	1/10	0	1/2	2/5	0	0
$\widetilde{58}$	-1/10	0	1/2	-2/5	0	0

IJ=$\frac{3}{2}$2	143	232	233	234	321	322	323	343	412	432
$\overline{27}$	0	1/18	5/18	0	0	0	0	0	2/5	2/45
$\overline{36}$	0	1/18	-5/18	0	0	0	0	0	2/5	2/45
$\overline{58}$	-1/10	0	0	0	1/10	0	2/5	-2/5	0	0
$\widetilde{36}$	-2/45	0	0	0	0	-1/2	-5/18	-8/45	0	0
$\overline{67}$	1/90	0	0	0	9/10	0	-2/45	2/45	0	0
$\widetilde{27}$	2/45	0	0	0	0	-1/2	5/18	8/45	0	0
$\widetilde{67}$	0	-2/9	0	5/18	0	0	0	0	1/10	-8/45
$\widetilde{58}$	0	-2/9	0	-5/18	0	0	0	0	1/10	-8/45

	433	434
$\overline{27}$	2/9	0
$\overline{36}$	-2/9	0
$\overline{58}$	0	0
$\widetilde{36}$	0	0
$\overline{67}$	0	0
$\widetilde{27}$	0	0
$\widetilde{67}$	0	2/9
$\widetilde{58}$	0	-2/9

IJ=$\frac{3}{2}$1	142	144	231	232	23_13	23_23	322	323	324	342
$\overline{27}$	2/45	0	0	0	0	0	5/18	-1/2	0	8/45
$\overline{36}$	2/45	0	0	0	0	0	5/18	1/2	0	8/45
$\overline{58}$	0	0	-1/18	0	2/9	-2/9	0	0	0	0
$\widetilde{36}$	0	0	0	5/18	10/81	5/162	0	0	0	0
$\overline{67}$	0	0	-1/2	0	-2/81	2/81	0	0	0	0
$\widetilde{27}$	0	0	0	5/18	-10/81	-5/162	0	0	0	0
$\widetilde{67}$	-1/18	2/45	0	0	0	0	2/9	0	-5/18	-2/9
$\widetilde{58}$	-1/18	-2/45	0	0	0	0	2/9	0	5/18	-2/9
$\overline{23}$	0	0	0	0	5/81	20/81	0	0	0	0
$\widetilde{23}$	0	1/9	0	0	0	0	0	0	4/9	0

TABLE 1d Cont-

	344	413	431	432	$43_1 3$	$43_2 3$
$\overline{27}$	0	0	0	0	0	0
$\overline{36}$	0	0	0	0	0	0
$\overline{58}$	0	1/10	-2/45	0	8/45	-8/45
$\widetilde{36}$	0	-2/9	0	2/9	8/81	2/81
$\overline{67}$	0	-1/90	-2/5	0	-8/405	8/405
$\widetilde{27}$	0	2/9	0	2/9	-8/81	-2/81
$\widetilde{67}$	8/45	0	0	0	0	0
$\widetilde{58}$	-8/45	0	0	0	0	0
$\overline{23}$	0	4/9	0	0	4/81	16/81
$\widetilde{23}$	4/9	0	0	0	0	0

IJ=$\frac{3}{2}0$	141	143	232	234	323	341	343	414	432	434
$\overline{58}$	-1/50	-2/25	0	0	1/2	-2/25	-8/25	0	0	0
$\overline{67}$	-9/50	2/225	0	0	-1/18	-18/25	8/225	0	0	0
$\widetilde{67}$	0	0	-5/18	2/9	0	0	0	-1/10	-2/9	8/45
$\widetilde{58}$	0	0	-5/18	-2/9	0	0	0	1/10	-2/9	-8/45
$\overline{23}$	0	1/9	0	0	4/9	0	4/9	0	0	0
$\widetilde{23}$	0	0	0	1/9	0	0	0	4/5	0	4/45

IJ=$\frac{1}{2}3$	233	322	433
$\overline{67}$	0	1	0
$\widetilde{67}$	-5/9	0	-4/9

IJ=$\frac{1}{2}2$	143	146	232	233	235	236	322	323	325	343
$\overline{18}$	0	0	1/72	1/8	-5/36	-2/9	0	0	0	0
$\overline{27}$	49/900	1/25	0	0	0	0	-1/16	49/144	-1/8	49/225
$\overline{36}$	-1/225	-1/25	0	0	0	0	-1/4	-1/36	-1/2	-4/225
$\overline{47}$	0	0	1/16	-1/16	-5/72	1/9	0	0	0	0
$\widetilde{36}$	0	0	1/36	-1/36	5/18	-1/9	0	0	0	0
$\overline{67}$	0	0	-4/9	0	0	0	0	0	0	0
$\widetilde{27}$	0	0	1/144	49/144	5/72	1/9	0	0	0	0
$\widetilde{47}$	-1/100	1/25	0	0	0	0	-9/16	-1/16	1/8	-1/25
$\widetilde{67}$	-1/9	0	0	0	0	0	0	4/9	0	-4/9
$\widetilde{18}$	1/50	-2/25	0	0	0	0	-1/8	1/8	1/4	2/25

	346	412	432	433	435	436
$\overline{18}$	0	1/10	1/90	1/10	-1/9	-8/45
$\overline{27}$	4/25	0	0	0	0	0
$\overline{36}$	-4/25	0	0	0	0	0
$\overline{47}$	0	9/20	1/20	-1/20	-1/18	4/45
$\widetilde{36}$	0	1/5	1/45	-1/45	2/9	-4/45
$\overline{67}$	0	1/5	-16/45	0	0	0
$\widetilde{27}$	0	1/20	1/180	49/180	1/18	4/45
$\widetilde{47}$	4/25	0	0	0	0	0
$\widetilde{67}$	0	0	0	0	0	0
$\widetilde{18}$	-8/25	0	0	0	0	0

TABLE 1d Cont-

IJ=$\frac{1}{2}$0	143	232	235	236	323	326	343	432	435	436
$\overline{23}$	0	0	-5/18	5/18	0	0	0	0	-2/9	2/9
$\widetilde{23}$	-1/90	0	0	0	-2/45	9/10	-2/45	0	0	0
$\overline{34}$	1/10	0	0	0	2/5	1/10	2/5	0	0	0
$\widetilde{34}$	0	0	5/18	5/18	0	0	0	0	2/9	2/9
$\overline{67}$	0	-5/9	0	0	0	0	0	-4/9	0	0
$\widetilde{67}$	-4/45	0	0	0	5/9	0	-16/45	0	0	0

IJ=$\frac{1}{2}$1	142	145	146	232	23_13	23_23	235	23_16	23_26	322
$\overline{18}$	1/90	-2/45	2/45	0	0	0	0	0	0	5/72
$\overline{27}$	0	0	0	5/144	-49/324	-49/1296	5/72	1/18	-1/18	0
$\overline{36}$	0	0	0	5/36	1/81	1/324	5/18	-1/18	1/18	0
$\overline{47}$	1/20	-1/45	-1/45	0	0	0	0	0	0	5/16
$\widetilde{36}$	1/45	4/45	1/45	0	0	0	0	0	0	5/36
$\overline{67}$	-1/9	0	0	0	0	0	0	0	0	4/9
$\widetilde{27}$	1/180	1/45	-1/45	0	0	0	0	0	0	5/144
$\widetilde{47}$	0	0	0	5/16	1/36	1/144	-5/72	1/18	-1/18	0
$\widetilde{67}$	0	0	0	0	20/81	-20/81	0	0	0	0
$\widetilde{18}$	0	0	0	5/72	-1/18	-1/72	-5/36	-1/9	1/9	0
$\overline{23}$	0	1/90	2/45	0	0	0	0	0	0	0
$\widetilde{23}$	0	0	0	0	-1/162	-2/81	0	-1/4	-1/4	0
$\overline{34}$	0	0	0	0	1/18	2/9	0	-1/36	-1/36	0
$\widetilde{34}$	0	-1/90	2/45	0	0	0	0	0	0	0

	323	325	326	342	345	346	413	432	43_13	43_23
$\overline{18}$	-9/40	1/36	2/45	-8/45	8/45	0	0	0	0	0
$\overline{27}$	0	0	0	0	0	49/180	1/36	-49/405	-49/1620	1/18
$\overline{36}$	0	0	0	0	0	-1/45	1/9	4/405	1/405	2/9
$\overline{47}$	9/80	1/72	1/5	-4/45	-4/45	0	0	0	0	0
$\widetilde{36}$	1/20	-1/18	4/45	16/45	4/45	0	0	0	0	0
$\overline{67}$	0	0	-4/9	0	0	0	0	0	0	0
$\widetilde{27}$	-49/80	-1/72	1/45	4/45	-4/45	0	0	0	0	0
$\widetilde{47}$	0	0	0	0	0	-1/20	1/4	1/45	1/180	-1/18
$\widetilde{67}$	0	0	0	0	0	1/9	0	16/81	-16/81	0
$\widetilde{18}$	0	0	0	0	0	1/10	1/18	-2/45	-1/90	-1/9
$\overline{23}$	0	4/9	0	2/45	8/45	0	0	0	0	0
$\widetilde{23}$	0	0	0	0	0	-2/45	0	-2/405	-8/405	0
$\overline{34}$	0	0	0	0	0	2/5	0	2/45	8/45	0
$\widetilde{34}$	0	-4/9	0	-2/45	8/45	0	0	0	0	0

TABLE 1d Cont-

	435	43_16	43_26
$\overline{18}$	0	0	8/45
$\overline{27}$	2/45	-2/45	0
$\overline{36}$	-2/45	2/45	0
$\overline{47}$	0	0	-4/45
$\widetilde{36}$	0	0	4/45
$\overline{67}$	0	0	0
$\widetilde{27}$	0	0	-4/45
$\widetilde{47}$	2/45	-2/45	0
$\widetilde{67}$	0	0	0
$\widetilde{18}$	-4/45	4/45	0
$\overline{23}$	0	0	-5/18
$\widetilde{23}$	-1/5	-1/5	0
$\overline{34}$	-1/45	-1/45	0
$\widetilde{34}$	0	0	-5/18

TABLE 1e

IJ=13	233	322	411	433
$\overline{68}$	-1/3	0	2/5	-4/15
77	2/9	0	3/5	8/45
$\widetilde{68}$	0	1	0	0

IJ=12	143	232	233	321	322	323	343	412	432	433
$\overline{28}$	0	1/36	5/12	0	0	0	0	1/5	1/45	1/3
$\overline{37}$	0	1/12	-5/36	0	0	0	0	3/5	1/15	-1/9
$\overline{68}$	-1/15	0	0	2/5	0	4/15	-4/15	0	0	0
$\widetilde{37}$	-1/45	0	0	0	-3/4	-5/36	-4/45	0	0	0
77	2/45	0	0	3/5	0	-8/45	8/45	0	0	0
$\widetilde{28}$	1/15	0	0	0	-1/4	5/12	4/15	0	0	0
$\widetilde{68}$	0	-4/9	0	0	0	0	0	1/5	-16/45	0

IJ=11	142	231	232	$23_{1}3$	$23_{2}3$	322	323	342	413	431
$\overline{28}$	1/45	0	0	0	0	5/36	-3/4	4/45	0	0
$\overline{37}$	1/15	0	0	0	0	5/12	1/4	4/15	0	0
$\overline{68}$	0	-2/9	0	4/27	-4/27	0	0	0	1/15	-8/45
$\widetilde{37}$	0	0	5/12	5/81	5/324	0	0	0	-1/9	0
$\widetilde{28}$	0	0	5/36	-5/27	-5/108	0	0	0	1/3	0
$\widetilde{68}$	-1/9	0	0	0	0	4/9	0	-4/9	0	0
77	0	-1/3	0	-8/81	8/81	0	0	0	-2/45	-4/15
33	0	0	0	5/81	20/81	0	0	0	4/9	0

	432	$43_{1}3$	$43_{2}3$
$\overline{28}$	0	0	0
$\overline{37}$	0	0	0
$\overline{68}$	0	16/135	-16/135
$\widetilde{37}$	1/3	4/81	1/81
$\widetilde{28}$	1/9	-4/27	-1/27
$\widetilde{68}$	0	0	0
77	0	-32/405	32/405
33	0	4/81	16/81

IJ=10	141	143	232	323	341	343	432
$\overline{68}$	-2/25	-4/75	0	1/3	-8/25	-16/75	0
77	-3/25	8/225	0	-2/9	-12/25	32/225	0
$\widetilde{68}$	0	0	-5/9	0	0	0	-4/9
33	0	1/9	0	4/9	0	4/9	0

IJ=03	322
77	1

IJ=02	232	235	322	325	412	432	435
$\overline{37}$	0	0	-1/2	-1/2	0	0	0
$\overline{48}$	1/18	-5/18	0	0	2/5	2/45	-2/9
$\widetilde{37}$	1/18	5/18	0	0	2/5	2/45	2/9
77	-4/9	0	0	0	1/5	-16/45	0
$\widetilde{48}$	0	0	-1/2	1/2	0	0	0

TABLE 1e Cont-

IJ=01	142	145	232	235	322	325	342	345	432	435
$\overline{37}$	0	0	5/18	5/18	0	0	0	0	2/9	2/9
$\overline{48}$	2/45	-4/45	0	0	5/18	1/18	8/45	-16/45	0	0
$\widetilde{37}$	2/45	4/45	0	0	5/18	-1/18	8/45	16/45	0	0
77	-1/9	0	0	0	4/9	0	-4/9	0	0	0
$\widetilde{48}$	0	0	5/18	-5/18	0	0	0	0	2/9	-2/9
33	0	1/45	0	0	0	8/9	0	4/45	0	0

IJ=00	232	235	432	435
77	-5/9	0	-4/9	0
33	0	-5/9	0	-4/9

TABLE 1f

IJ=$\frac{1}{2}$3	322	411					
$\overline{78}$	0	1					
$\widetilde{78}$	1	0					
IJ=$\frac{1}{2}$2	232	321	322	412	432		
$\overline{38}$	1/9	0	0	4/5	4/45		
$\overline{78}$	0	1	0	0	0		
$\widetilde{38}$	0	0	-1	0	0		
$\widetilde{78}$	-4/9	0	0	1/5	-16/45		
IJ=$\frac{1}{2}$1	142	231	232	322	342	431	432
$\overline{38}$	4/45	0	0	5/9	16/45	0	0
$\overline{78}$	0	-5/9	0	0	0	-4/9	0
$\widetilde{38}$	0	0	5/9	0	0	0	4/9
$\widetilde{78}$	-1/9	0	0	4/9	-4/9	0	0
IJ=$\frac{1}{2}$0	141	232	341	342			
$\overline{78}$	-1/5	0	-4/5	0			
$\widetilde{78}$	0	-5/9	0	-4/9			

TABLE 1g

IJ=03	411	
88	1	
IJ=02	321	
88	1	
IJ=01	231	431
88	-5/9	-4/9
IJ=00	141	341
88	-1/5	-4/5

TABLE 2

$\nu_1\nu_2$	ν	ϵ_2	$\nu_1\nu_2$	ν	ϵ_2	$\nu_1\nu_2$	ν	ϵ_2
[4] [11]	[51]	1	[22] [11]	[33]	1	[211] [11]	[2211]	1
	[411]	1		[321]	-1		$[21^4]$	1
[31] [11]	[42]	1		[2211]	-1	$[1^4]$ [11]	[2211]	1
	[321]	1	[211][11]	[321]	1		$[21^4]$	-1
	$[31^3]$	-1		[222]	-1		$[1^6]$	1
	[411]	-1		$[31^3]$	1			